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ŚRĪYANTRA AND ITS MATHEMATICAL PROPERTIES

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The *Śrīyantra* (the Great or Supreme *yantra*) is the ancient geometrical portrayal and it belongs to the class of objects (*yantras*) which are used for meditation in various schools of tāntrism. The process of precise reproduction of the *Śrīyantra* central seal (14-angled polygon composed by intersection of nine triangles) is a highly difficult problem which is above the capacity of hand-drawing because of requirement of precise superposition of numerous points of intersection. On the other hand, the task of general analysis of the *Srīyantra* seal is linked with such volume of calculations which is very far from the power of mordern computers.

The paper includes data of structural and analytical investigation of the *Śrīyantra* seal, the classification of available specimens and analysis of its frequency-and-time spreading. On this data is based the theory about the existence of ancient spherical origin of the *Śrīyantra* seal. In this case, the more simple and more contemporary specimens may have been produced by a century-old accumulation of errors in successive reproduction of this prototype. In the conclusion it is emphasized that a deeper study of this complicated and relatively unknown phenomenon requires co-operation of specialists from different fields of knowledge.

INTRODUCTION

From the ancient world we can find out examples of some cultural achievements, which, at first sight, may appear to have used very high mathematical knowledge much above the capacities of the ancient culture. The investigations of such phenomenons may lead us to discover the cultural and historical alternatives of the mathematical knowledge and help us to understand more deeply the significance of the world's scientific-and-technological progress. One of such unique objects is the *Srīyantra* (the Great or Supreme *yantra*) of Indian Tāntric Tradition. The mathematical properties of this *yantra* is very complicated, and its interpretation is linked with very deep cosmogonic and psychophysiological concepts.

The *Śrīyantra* belongs to the class of objects (*yantras*) which are used for meditation in various schools of tàntrism[1]. One of the earliest known specimens is the portrait of the *Śrīyantra* in the religious institution *Śṛṅgarī Maṭha* established by the famous philosopher Śaṅkara in eighth century A.D. The *Śrīyantra* had also been mentioned in the Buddhist inscription of the Śrīvijaya school in South Sumatra, which is dated seventh century A.D.[2] Therefore, the *Śrīyantra* already has covered a long path of confirmation as an important object for rituals. Thus, the hymn from *Atharva Veda*[3] (*c.* 12th century B.C.) is dedicated to the *Śrīyantra*-like figure composed of nine triangles.

The *Srīyantra* consists of a central 14-angles seal (polygon), 8 and 16-petalled lotus and the square of defence with four doors on four sides of the world (*bhūpura*). The seal is composed by intersection of nine big triangles, and, as a result of that, 43 small triangles are produced (Fig. 1).

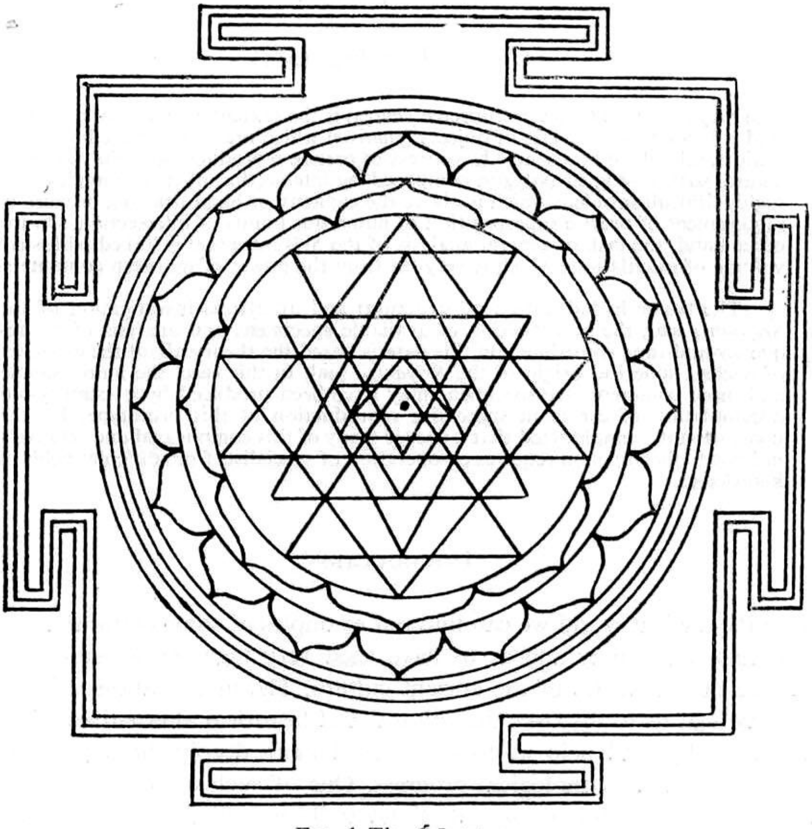

FIG. 1 The *Śrīyantra*.

There are two ways of the contemplation of the *Śrīyantra:* outward and inward, i.e. from the central point (*bindu*) outward to *bhūpura* through the sequence of enclosing circles of the small triangles, petals of lotus and lines, and in opposite direction. These two methods are in use among the followers of "right-hand-path" and "left-hand-path" sects of tantra. The outward sequence of contemplation is associated with the evolutionary development of the universe from the primordial out-of-time and out-of-space state (unity of Śiva and Śakti: the supreme conciousness and power, male and female principles) to phenomenal manifestation and to the more and more deep differentiation and complexity of matter. The inward sequence is associated with the process of destruction of the universe.

During meditation, the adept (*sādhaka*) imagines the projection of the evolutionary-involutionary processes inside his body, and, as a result of that, the power of Śakti (called *Kuṇḍalinī*) awakes. This power is sleeping at a base of spinal column (in so called *Cakra Mulādhara* assosiated with the *bhūpura* of the *Śrīyantra*). The adept tries to send it upward to merge with Śiva's aspect resided at the head's *Cakra* (*Sahashara* associated with *bindu* of the *Śrīyantra*). According to the tāntric concepts, this process leads to indescribable increase of consciousness.

ANALYSIS

Our attention in following the paper is directed only[4] to the geometrical structure of the *Śrīyantra* seal. The process of reproduction of the *Śrīyantra* is turned out to be unexpectedly a very difficult problem. Most of the seal's lines pass through 3-6 points of intersection of other lines, and a lot of redrawing of the whole figure is needed in order to attain precise super position of points. Let us try to understand the very nature of this problem.

The structure of *Śrīyantra* seal geometrically consists of four enclosed components (Fig 2). The process of construction of each component includes the drawing of closed sequence of lines, in conjunction of which, it is needed to draw a line from three points of intersection of previously drawn lines.

Let us try to understand this process by construction of the component I. (Fig. 2*a*). First of all let us choose *a priori* value yA (it will be corrected by construction of component 4) and draw the two largest and symmetrical triangles (thin lines). Then let us arbitrarily place point A′ at horizontal line, draw the line connecting the points 1, A' and then draw horizontal line from point 2. Now we have three points 3, A', D, and we must draw line through points 3, A', so as to meet the point D. In order to attain this goal, a sequence of approximations (the iterative process in mathematical sense) is necessary to perform. The construction of component 2 is almost in symmetry with component 1 (the value yB chosen *a priori* will be corrected by construction of component 3). When components 1, 2 are drawn, we have all lines needed to start the construction of component 3. After that the construction of component 4 may also be performed.

Therefore, the co-operation among four iterative procedures has three levels of enclosure (Fig. 3). At each step of approximation for third component it is needed to perform a full cycle of iterative steps for second component. And at each iterative step for fourth component the first three approximation processes must be completely fulfilled. Thus, if we designate the amount of iterative steps needed to attain determined precision of drawing for components 1-4, correspondingly, with letters a, b, c, d, then the painting of the *Śrīyantra* seal may be expressed by following formula:

$$N = ad + bcd + cd + d \qquad (1)$$

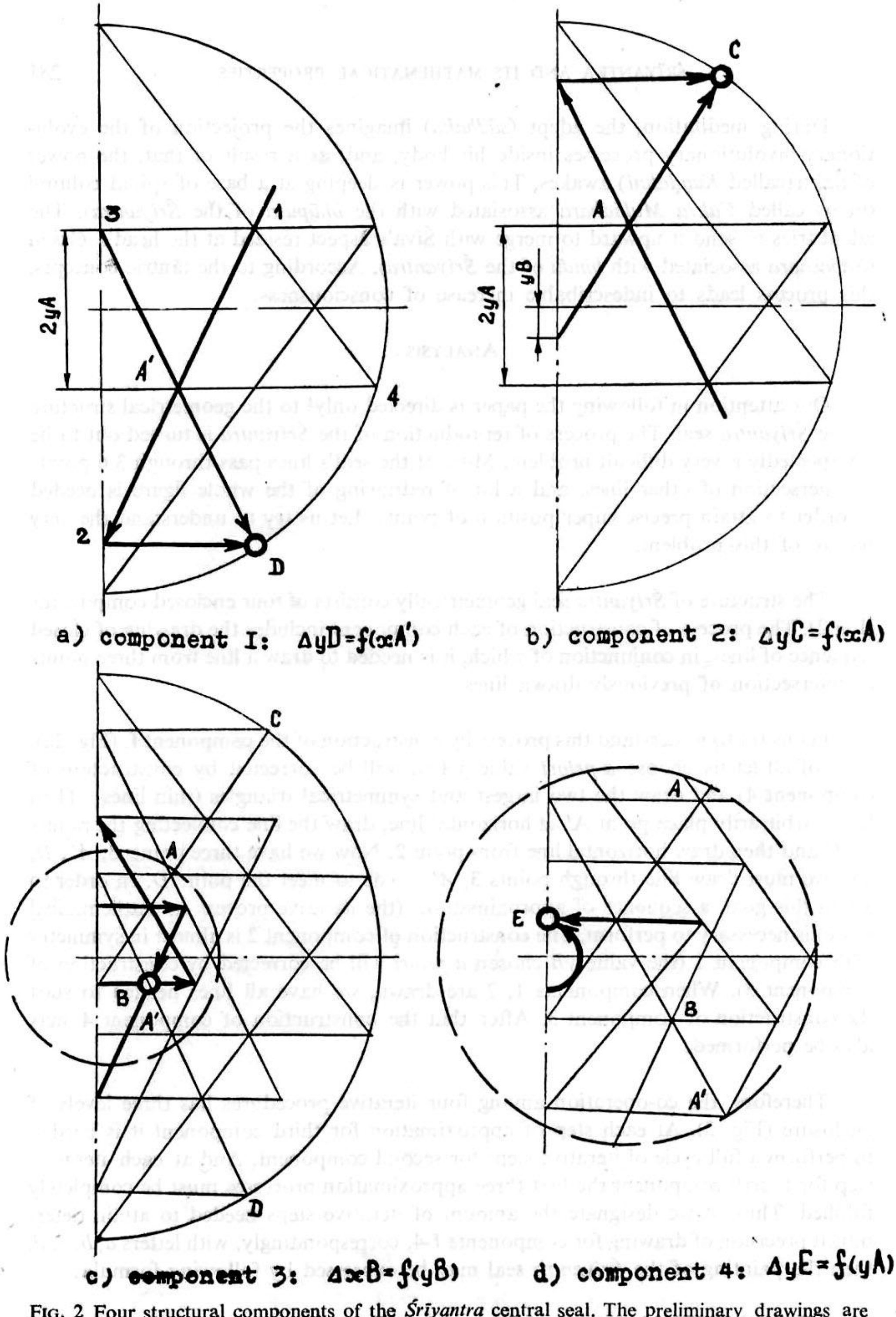

FIG. 2 Four structural components of the *Śrīyantra* central seal. The preliminary drawings are marked by thin lines, the main drawings are marked by heavy lines. The directions of arrows show the chosen order of construction. The functions for the appreciation of precision of construction are put below the schemes.

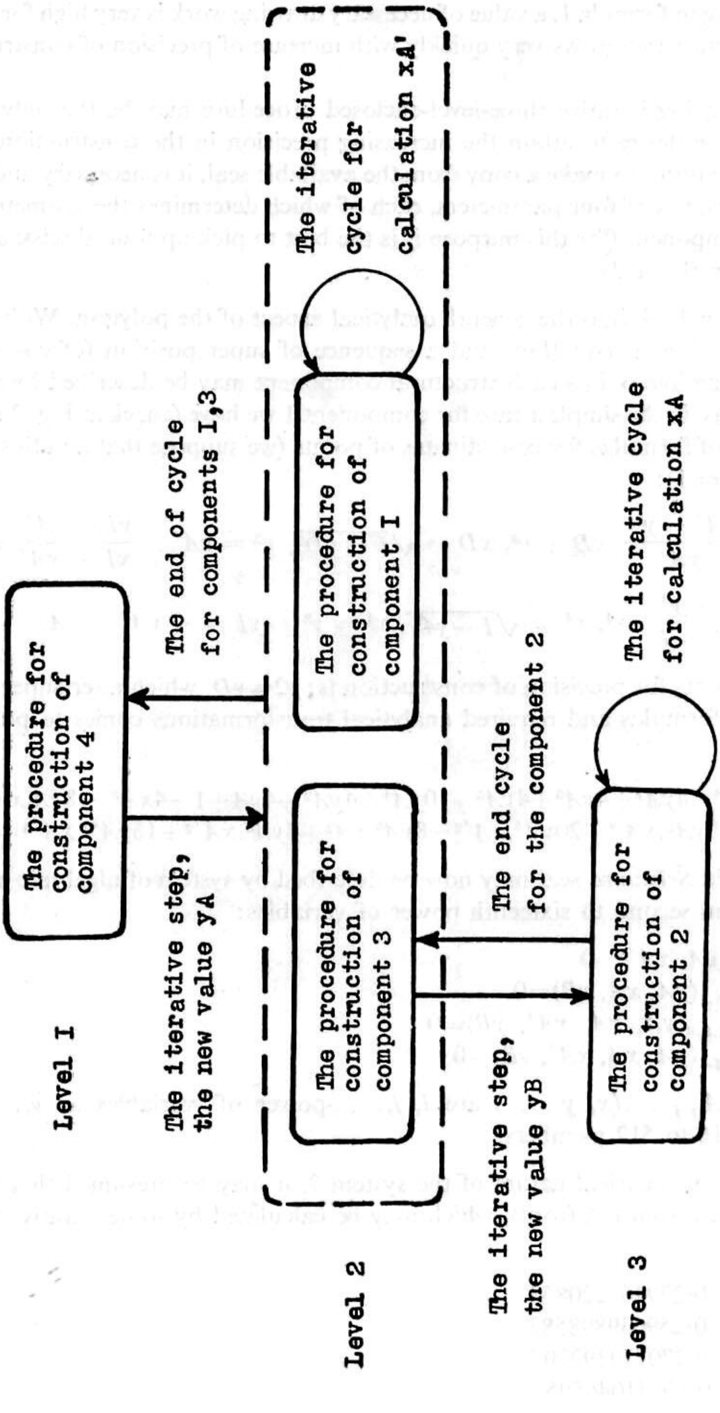

FIG. 3 The scheme shows the interaction between the iterative construction procedures for four components of the *Śrīyantra* seal.

According to formula 1, a value of necessary drawing work is very high for ordinary human capacities and grows very quickly with increase of precision of construction.

The foregoing iterative three-level-enclosed procedure may be the only possible technique if we desire to attain the increasing precision in the construction. On the other hand, in order to make a copy from the available seal, it is necessary and enough to know the values of four parameters, each of which determines the geometry of one structural component (for this purpose it is the best to pick up four abscissae of horizontal lines in the seal).

Let us now look into the general analytical aspect of the polygon. With the help of linear and circular equations and a sequence of super position (of co-ordinates) of the matching line points each structural component may be described by algebraic equation. Thus, in the simplest case for component 1 we have (check at Fig. 2*a*) following sequence of formulas for co-ordinates of points (we suppose that a radius of outer circumference=1):

$$yD = \frac{yA' - y^3}{yA'} \cdot xD + y^3,\ xD = \sqrt{1 - yD^2},\ y^2 = yA' - \frac{y1 - yA'}{x1 - xA'} \cdot xA'$$

$$x1 = \frac{1 - y^4}{y^4 + 1} \cdot x^4,\ x^4 = \sqrt{1 - y4^2},\ y^4 = y^3 = y1 = -yA' = yA.$$

The requirements for precision of construction is: $y2=yD$, which after super position of foregoing formulas and required analytical transformations comes to polynomial form:

$$P=yA^8-4yA^7+4yA^6+4yA^5-10yA^4+4yA^2-4yA+1-4xA^2+8yA.xA'^2$$
$$-20yA^2.xA'^2+20yA^4.xA'^2-8yA^5.xA^2+4yA^6.xA'^2+16yA'^2.xA'^4=0$$

The whole *Śrīyantra* seal may now be described by system of algebraic nonlinear equations from second to sixteenth power of variables:

$$\begin{aligned} &P_{8.4}(yA, xA')=0\\ &Q_{14.8.4}(yA, xA, yB)=0\\ &R_{16.4.6.6}(yA, xA, xA', yB)=0\\ &S_{6.3.2.4}(yA, xA, xA', yB)=0 \end{aligned} \qquad (2)$$

Polynomials $A_{i,j,\ldots}(x, y, \ldots)$ are $i, j, \ldots$-power of variables $x, y, \ldots$ and consist from 16 to 512 members.

From the geometrical nature of the system 2, it may be presumed that there is, at least, one real solution (root), which may be calculated by some iterative method. Such root is:

$$\begin{aligned} yA &= 0{\cdot}279461220858\\ xA &= 0{\cdot}259039898582\\ xA' &= 0{\cdot}270779392707\\ yB &= -0{\cdot}10141046595 \end{aligned} \qquad (3)$$

But, at the same time, system 2 may or may not possess other real roots. The modern theory (higher algebra) does not give a direct answer to this question. The application of the iterative methods for finding of appropriate values of all roots requires the knowledge of bounds of its localisation. This is solved only for the polynomial of a single variable. By applying the process of consecutive exclusion of variables system 2 may theoretically lead to a polynomial of not higher than 12544 power of a single variable. However, the volume of required analytical transformations is boundless even for the high-power computers. Thus, for first step of transformation (from four required ones) computer must perform more than 10^{11} operations (in case of very thorough programming), and a volume of calculations on each succeeded step, at least, in 100 times more than preceding one. Moreover, the investigation of such resulting equation requires to operate with a number representation of 4 thousand figures. These demands very far exceed the capacities of modern computers.

This conclusion induces the following questions to arise: "What sort of representational means and knowledge had been used for reproduction of the *Śrīyantra* seal during centuries?" and "How the idea of the nine triangles consolidating in such polygon with precise matching of numerous points of intersection arose?". For the answer, first of all, it is necessary to investigate the in-time-and-in-space spreading of different specimens of the *Śrīyantra*. Let us try, as it is possible, to solve up these problems on the basis of few numerous available original materials.

Among available tāntric specimens of the *Śrīyantra*, three types of portrayals can be distinguished[5] (Fig.4). The first is the most widespread one (Fig.4a) and is characterized by the components 1,2 freely disposed, because the angles C,D of the triangles do not belong to circumscribed circumference. On account of that, the first two iterations are not required and the difficulty of construction may be minimized to: $N \leqq 2\cdot c\cdot d+2\cdot d$ (compare with formula 1). The task would be simplified further if we throw back the demand of co-axle of outer circumscribed circumference and the inner inscribed one. In that case, the iterative cycle for the component 4 is also eliminated, and the difficulty is diminished to the perfectly acceptable value: $N=2\cdot d$. This circumstance, obviously, is the main reason for the wide spreading of I-type protrayals.

For this type the method of traditional copying[6] is well-known, according to which (Fig.5), the vertical diameter is divided into 48 equal parts, after that the horizontal lines of the polygon are drawn on the levels of subdivisions of 6,12,17,20,23,27,30,36, 42. And, as an accomplishment of that, the last lines are drawn through the available points. However, this "heuristic" method even for so simple portrayal, does not ensure (even for a visual perception) the satisfactory matching of some points of intersection.

According to the opposite of copying scheme[7], the polygon of the *Śrīyantra* is constructed by continuously lengthening the sides of triangles, beginning from the innermost triangle (Fig.6). But possibility of realization of this scheme is doubtful[8].

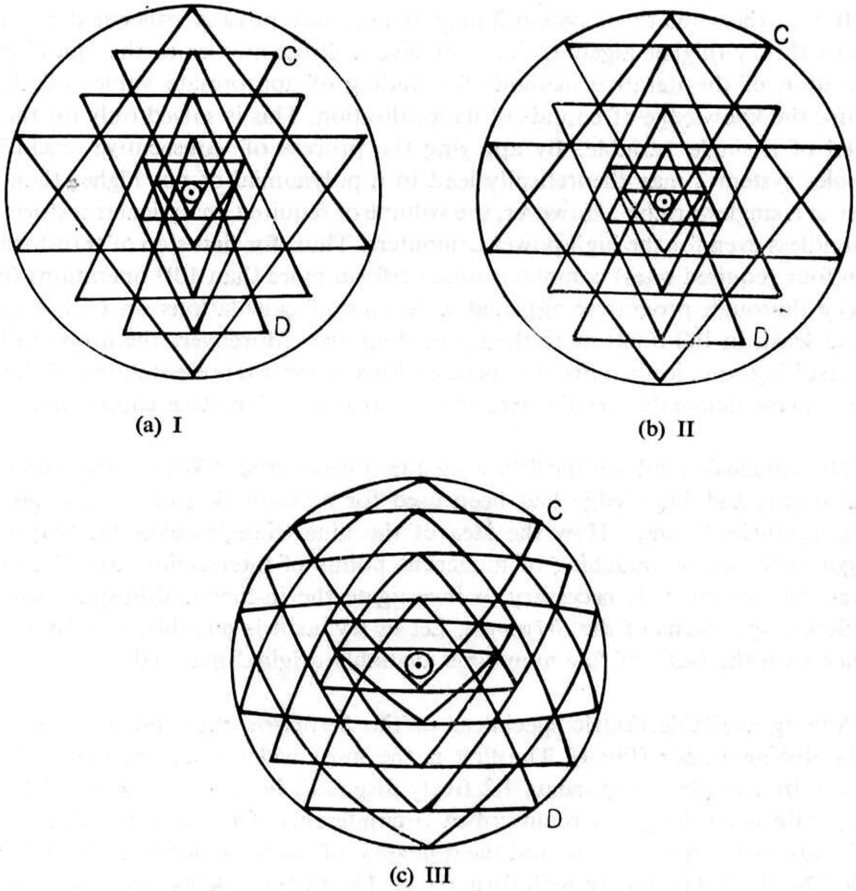

FIG. 4 Examples of three types of the *Śrīyantra* seal.

Indeed, in that case, just at the third step of drawing, the amount of the point (co-ordinates) chosen *a priori* becomes more than four, i.e. the number of independent seal's parameters (as it is shown here). Moreover, an inaccuracy in the previously chosen points has an increasing effect on the further drawing. Therefore, this construction may be in the nature of very symbolic one.

As it may be noticed, neither of these two instructions allow us to directly increase the precision of construction in opposition to the above-mentioned iterative process.

The second type of the *Śrīyantra* seal (Fig.4b) is spread less than the first one, and it is characterized by the angle point *C* of component 2 belonging to outer circumference, on account of that, the difficulty of drawing increases to: $N=b.c.d+c.d+2.d$, so it nearly reaches the utmost value (cf. formula 1). The instruction for drawing of this type of portrayals is not known.

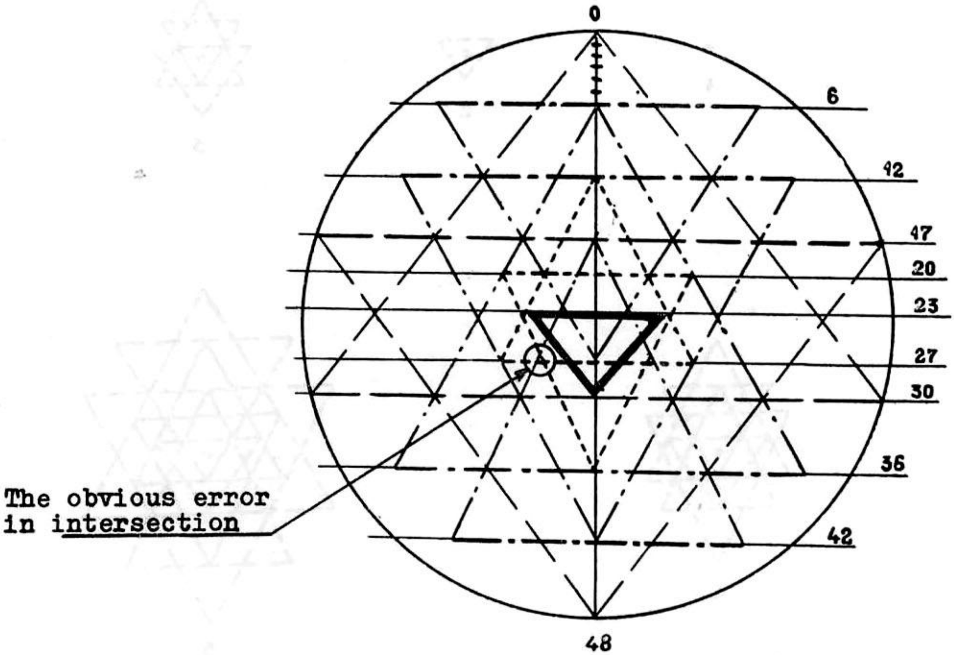

FIG. 5 The traditional method of rough reproduction of I-type *Śrīyantra* seal in the order of destruction (*saṃhāra-krama*) in such sequence:

1) ————————

2) — — — —

3) —·—·—

4) - - - - - - - - - - - -

5) ━━━━━━━━

The third type of the portrayals (Fig.4c) is found very rarely and is characterized by the points C,D of the component 1,2 strictly fixed at the outer circumference and by employment of arcs of ovals or ellipses instead of lines. therefore, these portrayals may be the plane projection of the spherical image (it may be added that some originals are drawn on noticable convex surface[9]).

At the spherical construction (Fig.7) the outer circumference of the polygon is formed by the side cross-section, and its position may be determined by an angle α comparativly to the sphere's axis Z, which is perpendicular to the crossplane. The sides of triangles are the geodesical lines on the sphere (they are formed by the central cross-sections). If $\alpha \rightarrow 0$, the spherical image transforms to its linear limit—III_1 (represented at Fig.1). It is significant that some real tāntric II-type specimens are near (because of point D is near to outer circumference) to this special linear case of spherical representation (Fig.8).[10]

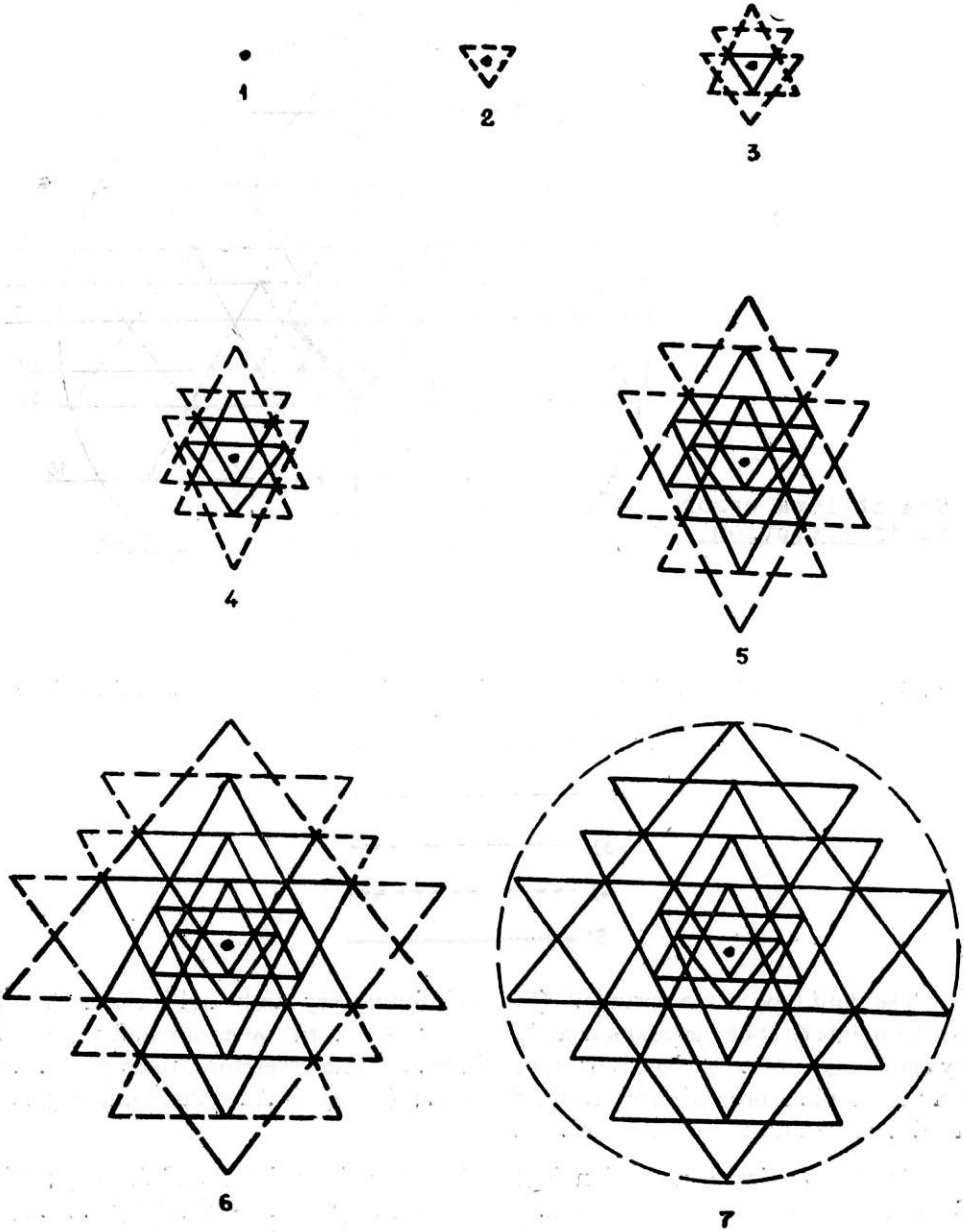

FIG. 6 The traditional method of very rough reproduction of I-type *Śrīyantra* seal in the order of creation (*sṛṣṭi-krama*).

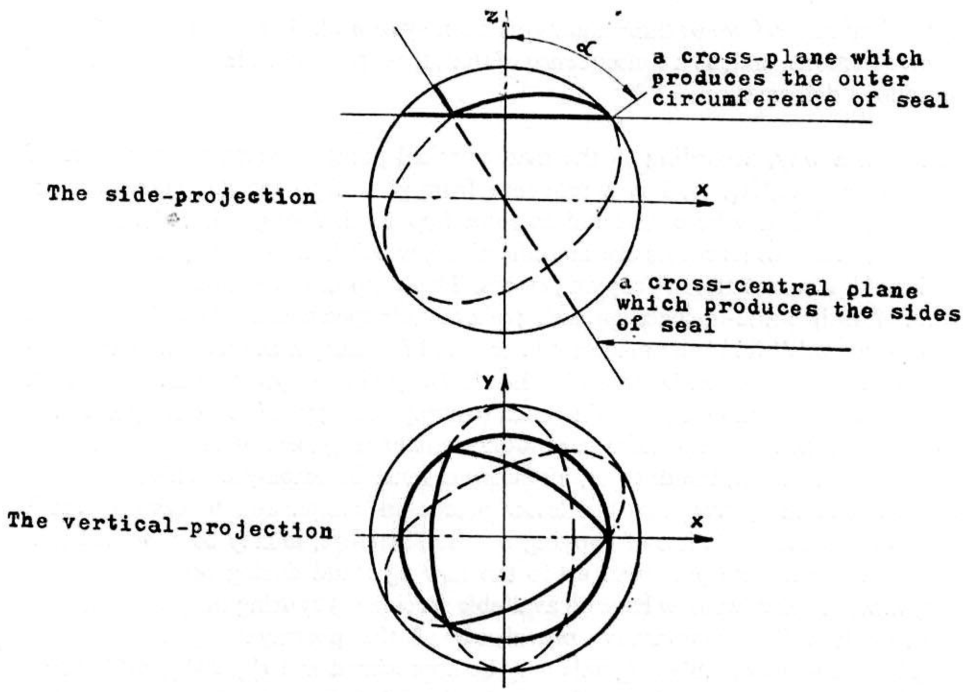

FIG. 7 The design of the spherical construction of the *Śrīyantra* seal.

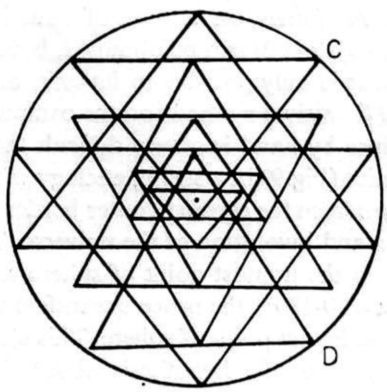

FIG. 8 The specimen of II-type seal is near-by to the plane limits of III-type seal.

The portrayals III and III_1 are described by a complete system of equations (just as system 2) including four equations from four variables. Therefore, its solutions are one or the discrete multitude of roots, on account of which these portrayals are rigid and undeformable. The portrayals type I and II are described by incomplete systems of equations (two or three equations from system 2). Therefore, its solutions are some functions, and, as a consequence of that, these portrayals have the chances of a continuous deformation.

In such a way, according to the mathematical point of view, the sequence of portrayals type III,III_1, II, I is a sequence from general to special. Therefore, the portrayal type III may be considered as some hypothetical original, and the others may be considered as its successive simplifications, which had sprung up as a result of centuries-old accumulation of copying errors. This supposition is in accordance with the data of in-time-and-in-space spreading of available specimens. Thus, three known III-type portrayals[11] had been made not later than 17 century A.D. on metal plate (produced by casting) or stone. On the other hand, the specimens type I, II have spread in more wide and contemporary time-intervals (correspondingly, 17-19 century A.D. and 18-20 century A.D.). These paintings are found mostly on papers or materials, and, in order to produce them, the drawing instruments must be employed. This method of reproduction is easily available but lesser precise in comparison to casting, and it promotes more accumulation of copying errors. Therefore, it may be supposed that III-type specimens were preserved up to the new age (and during period before 17 century A.D., in which we now have no available specimens) by using the precise matrix-copying method. The comparative prevalence of the portrayals type III, II, I in multitude of 30-40 available originals may be appreciated as 10 %, 20 %, 70 %. These data point to the more prevalence of more simple and more contemporary portrayals.

An algorithmical complexity of the spherical representation of the *Śrīyantra* seal is very much noticeable in comparison to the plane one, because of employment of second-power curves instead of lines, on account of that, it is doubtful if its polynomial description (similar to system 2) can be obtained, becuase of the high amount of analytical transformation. It is only possible to investigate a separate root of this representation by calculative (iterative) methods on the computer with the aid of computer's graphic (to draw ellipse by hand is a very difficult task). It is interesting that available root has the dynamics (Fig.9) in a strong analogy with the sequence of ritual contemplation of the *Śrīyantra* from the centre to outer border and otherwise, which is associated with the evolution and involution of the universe. Thus, during increase of an angle α, the seal rises from the upmost point of sphere and spreads on its upper surface until $\alpha=90°$. Then ($\alpha=90$-$180°$), the process transfers to its symmetrical opposition, i.e. the tightening to the lowest point of sphere. This characteristic of the spherical representation is a support to the hypothesis about the prototype III of the *Śrīyantra*.

The complexity of the construction of the *Śrīyantra* seal may guide us to an idea about an existence of some simple correlations in the polygon's structure. But in the investigation of III_1-type seal we have not found out the correlations expressed in whole numbers, vulgar fractions or well-known transcendentic constants. On the other hand, for the types I and II of portrayals, owing to its ability to continuous deformation, if

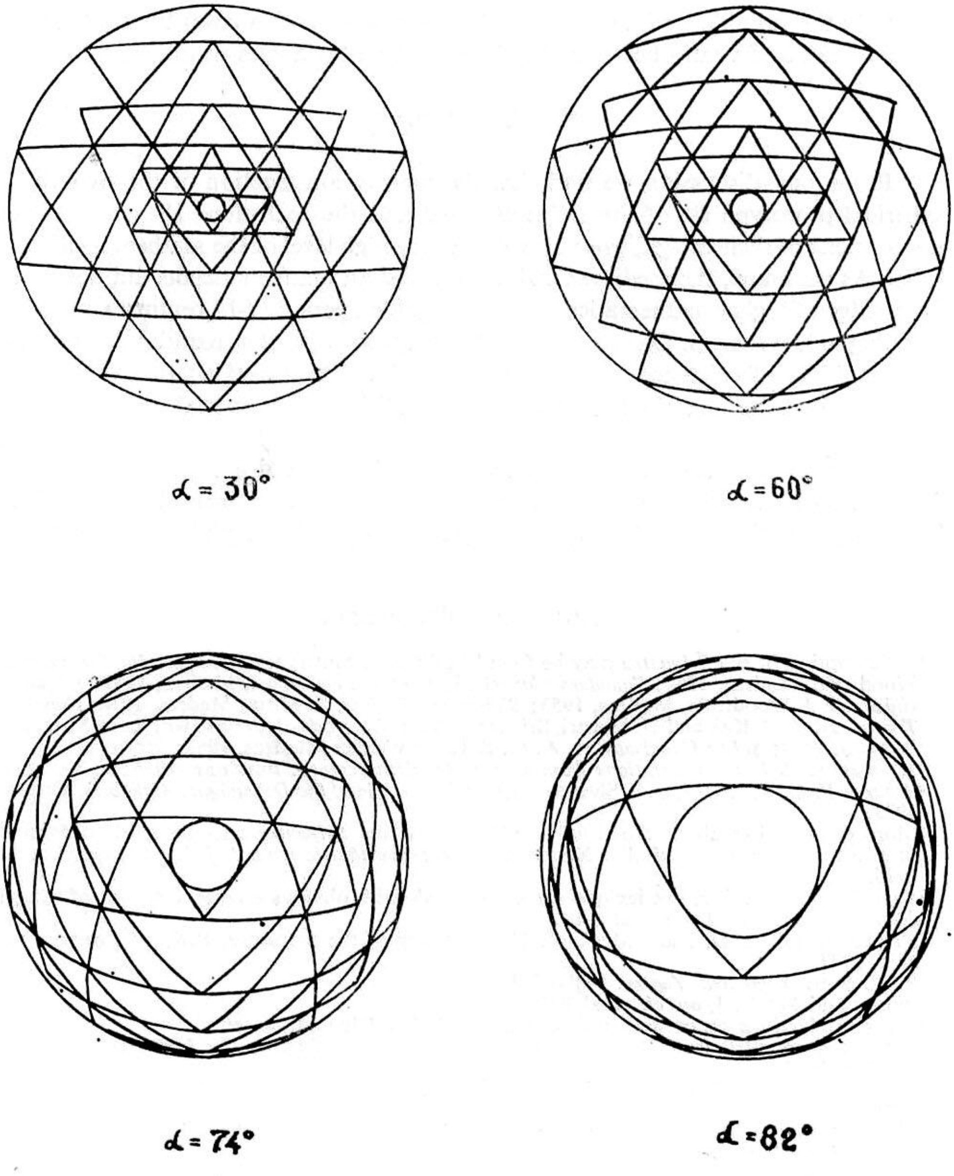

FIG. 9 The dynamics of III-type seal in accordance to a changing of an angle α, which determines the position of the outer circumference.
On these pictures the diameters of outer circumferences are equal to each other by appropriate choice of scale.

we apply the suitable limitation on the loose-parameters (on co-ordinates C,D), any required value of a ratio between the chosen sides of triangles would be achieved[12].

CONCLUSION

From our discussion we find that the precise construction of the hypothetical spherical prototype III of the *Śrīyantra* seal and the real plane II-type specimens, not to speak of their design, would involve a very high level of the mathematical knowledge. As we know, the medieval and ancient Indian mathematicians did not possess knowledge of higher mathematics, even at its golden period (7-12 century A.D.) of outstanding achievements. One of the possible ways to solve this paradox is to suppose the possibility of existence of unknown cultural-and-historical alternative of mathematical knowledge, e.g. the highly developed tradition of the special imagination.

The *Śrīyantra*, as shown here, is a very complicated and many-sided object, and for its deep study it is required to apply efforts by specialists from different fields of knowledge: mathematics, history, ethnography, psychology, philosophy, etc.

NOTES AND REFERENCES

[1]A description of the *Śrīyantra* may be found in different tāntric texts: *Tantrarāja Tantra*. ed. J. Woodroffe, Madras, 1954; *Soundarya-laharī* ed. Anantakṛṣna Sāstrī, Madras, 1957; *Kāmakalāvilāsa*. ed. J. Woodroffe, Madras, 1953; *Bhāvanopaniṣad*. tr. S. Mitra, Madras, 1976; *Gandharva Tantra*. ed. R. C. Kak and H. Shastri, Srinagar, 1934; *Nityaṣoda-sīkarṇava*. with com. Sivananda, Banaras, 1968; *Sākta Upanisads*. tr. A. G. Krishna Warrier, Madras, 1967; and others.

[2]J. Casparic. *Selected inscriptions from seventh to ninth century*. Bundung, 1956; p. 30, 34, 41.

[3]*Atharva Veda*, X, v. 31-4, tr. S. Shamasastry in: *The Origin of the Devanāgarī Alphabets*, Varanasi, 1973.

[4]More deep notion about the ritual significance of the *Śrīyantra* may be received from the excellent monograph by Madhu Khanna: *Yantra: The tāntric symbol of cosmic unity*. London, 1975.

[5]This classification does not include some portrayals with obvious errores, e.g. tops of triangles do not connect with horizontal lines.

[6]Nicolas J. Bolton, D. Nicol Macleod. The geometry of the *Śrīyantra*, *Religion*, London, v. 7, N 1, 1977.

[7]P. H. Pott. *Yoga and Yantra*. Nijhoff, 1966.

[8]Bolton and Macleod, *op. cit.*, and Pott, *op. cit.*

[9]The reproductions on convex surface, see in: Madhu Khanna, *op. cit.*

[10]Fig 8 is represented from the specimen of Philip Rawson, *Tantra. The Indian cult of ecstasy*. London, 1973.

[11]These three portrayals are in: Madhu Khanna, *op. cit.* and Nic Douglas. *Tantra yoga*. New Delhi, 1971.

[12]Similar attempts (in respect to golden proportion and number π) have been undertaken in: Bolton and Macleod, *op. cit.*, which is the only, we know, research of the *Srīyantra* as a mathematical object.